LOCAL PULSATIONS AND PHASE INTERACTION IN SUSPENSIONS OF FINE PARTICLES

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This paper gives expressions for the mean values of the quantities characterizing random psuedo-turbulent pulsations in a monodisperse suspension of solid spherical particles at small Reynolds numbers. It discusses the effect of these pulsations on the effective hydraulic resistance exerted by the particles on the carrying stream of liquid. The parameters characterizing the random chaotic motion (psuedo-turbulence) of the phases in locally homogeneous flows of a suspension of fine particles have been calculated earlier in [1, 2]. However, the relationships obtained in these articles are in need of a certain degree of refinement; this is bound up with the refinement [3] of certain results of the spectral theory of the concentration of disperse systems which were used in [1, 2], and with a more correct determination of the function characterizing the effect of the constriction of the flow around the particles on the force of the viscous interaction of each of the particles with the liquid. Such a refinement is proposed in the present article. The formulas presented below can be used in evaluation of the coefficients of psuedo-turbulent transfer, of the fluctuations of the velocities of the liquid and the particles, and of other quantities characterizing the local structure of a suspension.

1. We shall formulate the principal postulations used in what follows. First of all, we assume that the Reynolds number, which describes constricted flow around individual particles, is small. In this case, the flow of the suspension is locally homogeneous in the sense that, in the flow, there is no formation of large-scale fluctuations of the concentration, and the particles themselves may be regarded as statistically independent, neglecting their indirect collisions [1, 2]. In addition, we assume that the scales of the change in the mean values of the quantities characterizing the mean motion of the suspension exceed by far the corresponding scales of the pseudo-turbulent pulsations. This permits regarding the pseudo-turbulence as "equilibrium," i.e., neglecting the effect of the derivatives of the above quantities with respect to the coordinates and the time on the pulsations. In the limit, when these derivatives are equal to zero, the motion may be interpreted simply as the filtration of a liquid through a cloud of particles which, on the average, are motionless at a certain constant rate, $\mathbf{u}_f$, and this can be regarded as the simplest model of a fluidized bed. In this case, the equations for the mean motion have the form

$$\begin{gathered}(1-\rho)(-\nabla p+d_0\mathbf{g})-d_0\beta\rho K(\rho)\mathbf{u}=0,\\ \mathbf{u}_f=(1-\rho)\,\mathbf{u}\\ -\nabla p+d_1\mathbf{g}+d_0\beta K(\rho)\mathbf{u}=0,\quad \beta=9\nu_0/2a^2,\ \nu_0=\mu_0/d_0\end{gathered}\tag{1.1}$$

Here d_0, μ_0 are the density and the viscosity of the liquid; a, d_1 are the radius and the density of material of the particles; $\mathbf{g}$ is the acceleration due to gravity; ρ is the volumetric concentration of particles in the system, while the function $K(\rho)$ describes the deviation of the effective viscous force acting on a particle, under constricted flow conditions, on the Stokes force; $K(0)=1$. For simplification, we neglect also the effect of viscous stresses in the liquid phase on the development of pseudo-turbulent pulsations; as is clear from [1, 2], this does not lead to any substantial change in the results obtained.

With the above assumptions, we can write the following relationships for the spectral measures of the pseudo-turbulent pulsations of the pressure p', the velocities of the liquid $\mathbf{v}'$, and the particles $\mathbf{w}'$,

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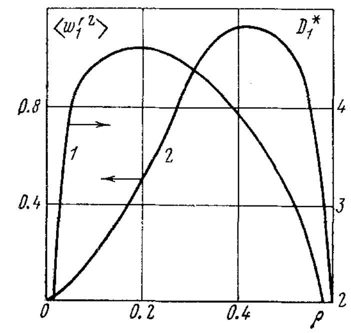

Fig. 1

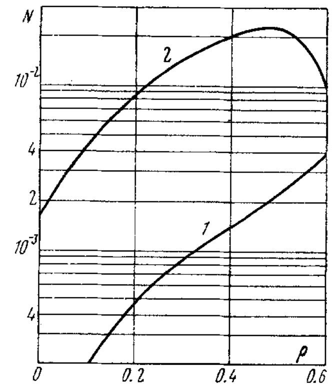

Fig. 2

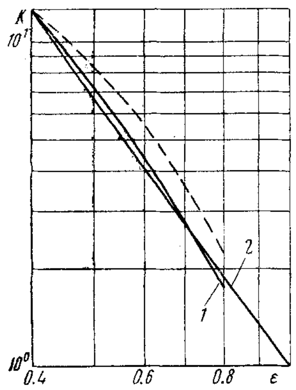

Fig. 3

and the concentration ρ', which figure in the representations of the above pulsations in the form of Fourier–Stieltjes integrals [2]

$$dZ_p = 0, \qquad d\mathbf{Z}_v = \frac{\omega + \mathbf{uk}}{1-\rho}\,\frac{\mathbf{k}}{k^2}\,dZ_\rho, \tag{1.2}$$
$$d\mathbf{Z}_w = \left(\frac{d\ln K}{d\rho}\,\mathbf{u} + \frac{\omega+\mathbf{uk}}{1-\rho}\,\frac{\mathbf{k}}{k^2}\right)dZ_\rho$$

where ω and $\mathbf{k}$ are the frequency and the wave vector. Relationships (1.2) permit expressing the different spectral densities characterizing psuedo-turbulent motion in terms of the spectral density $\Psi_{\rho,\rho}$ of a random process, ρ', and then, using the determined representation for $\Psi_{\rho,\rho}$, finding the correlation functions by the standard method (the expanded expressions for the spectral densities are given in [2]).

As $\Psi_{\rho,\rho}$, we here use the function [3]

$$\Psi_{\rho,\rho}(\omega, \mathbf{k}) = \frac{\mathbf{kDk}}{\pi}\,\frac{\Phi_{\rho,\rho}(\mathbf{k})}{\omega^2 + (\mathbf{kDk} - T_0\omega^2)^2}, \qquad T_0 = \frac{\operatorname{tr}\mathbf{D}}{\langle w'^2\rangle} \tag{1.3}$$
$$\Phi_{\rho,\rho}(\mathbf{k}) = \frac{3}{4\pi}\,\frac{\rho^2}{k_0^3}\left(1 - \frac{\rho}{\rho_*}\right)Y(k_0 - k), \qquad k_0 = \left(\frac{9\pi\rho}{2}\right)^{1/3}\frac{1}{a}$$

where $\mathbf{D}$ is the tensor of the coefficients of the psuedo-turbulent diffusion of the particles; ρ_* is the concentration of the particles in the densely packed state; $Y(x)$ is a Heaviside function.

To determine $K(\rho)$ we use the results of experiments on determination of the hydraulic resistance of a bed of motionless particles. We here assume that, in such experiments, there is usually determined the quantity $K_e(\rho)$, entering into the equation

$$-\nabla p + d_0\mathbf{g} - d_0\beta\rho K_e(\rho)\mathbf{u}_f = 0 \tag{1.4}$$

Comparing (1.4) with the first equation of (1.1), we have

$$K(\rho) = (1-\rho)^2 K_e(\rho) \tag{1.5}$$

For determinacy we use the empirical formula of Ergun [4]

$$K_e(\rho) = {}^{25}/_{3}\,\rho\,(1-\rho)^{-3} \tag{1.6}$$

which holds with $\rho > 0.2$–0.3. To extend the expression for $K(\rho)$, following from (1.5), (1.6), to the region $\rho < 0.2$–0.3, we further approximate this expression by the function

$$K(\rho) = \frac{2.96}{(1-\rho)^{1.73}} - 1.96 \tag{1.7}$$

The function (1.7) approximates the above expression sufficiently well at $\rho > 0.2$–0.3 and, at $\rho = 0$, reverts to unity. We use it also in calculations of the mean pseudo-turbulent values below; for ρ_* we use the value $\rho_* = 0.6$

2. The "equilibrium" pseudo-turbulence under consideration is obviously axisymmetric; the axis of symmetry is directed along the vector **u**. Selecting the coordinate axis x_1 along **u**, and using the method of [1], for the coefficients of the pseudo-turbulent diffusion of the particles, we obtain the equations

$$D_1 = \frac{1+\gamma^2}{\gamma k_0}\frac{\rho}{1-\rho}\left(\frac{3}{4}\right)^{1/2}\left(1-\frac{\rho}{\rho_*}\right)^{1/2}(J_2-J_4)^{1/2} au \tag{2.1}$$

$$D_2 = D_3 = N_D D_1 = \frac{\gamma^2}{1+\gamma^2}D_1, \qquad J_n = \int_0^1 \frac{t^n dt}{t^2+\gamma^2}$$

here γ is the sole root of the equation

$$\frac{2\gamma^2}{1+\gamma^2} = \frac{J_2-J_4}{l^2 J_0 + 2lJ_2 + J_4}, \qquad l = (1-\rho)\frac{d\ln K}{d\rho} \tag{2.2}$$

The dependences of the quantities $D_1^* = D_1/au$ and N_D on ρ, calculated in accordance with (2.1) and (2.2), are shown in Fig. 1 and Fig. 2, respectively (curves 1). The value of D_1^* is found to be less, and the value of N_D greater than the values calculated in [1].

For the mean squares of the components of the pulsation velocity of the particles, on the basis of (1.2), and (1.3) we use the usual method to obtain the expressions

$$\langle w_1'^2\rangle = \varphi\left[l^2 + \frac{2}{3}l + \frac{1}{5}\left(1+\frac{1+{}^2/_3 N_D}{1+2N_D}\frac{\langle w'^2\rangle}{u^2}\right)\right]u^2$$

$$\langle w_2'^2\rangle = \langle w_3'^2\rangle = N_w\langle w_1'^2\rangle = \frac{\varphi}{15}\left(1+\frac{1+4N_D}{1+2N_D}\frac{\langle w'^2\rangle}{u^2}\right)u^2 \tag{2.3}$$

$$\langle w'^2\rangle = \varphi\left(l^2+\frac{2}{3}l+\frac{1}{3}\right)\left(1-\frac{\varphi}{3}\right)^{-1}u^2, \qquad \varphi = \left(\frac{\rho}{1-\rho}\right)^2\left(1-\frac{\rho}{\rho_*}\right)$$

The dependences of $\langle w_1'^2\rangle^* = u^{-2}\langle w_1'^2\rangle$ and N_w on ρ, obtained from (2.3) at $N_D \approx 0$, are also shown in Fig. 1 and Fig. 2 (curves 2). These values are very close to those calculated in [2] over almost the whole region $0 < \rho < 0.6$, but they do not revert to infinity in the neighborhood of $\rho = 0.6$. The distribution functions of the quantities $\langle w_j'^2\rangle$ in the "equilibrium" state under consideration are Gaussian functions with variances following from (2.3).

Finally, we give expressions for the other pseudo-turbulent characteristics which are of interest. We have

$$\langle v_1'^2\rangle = \frac{\varphi}{5}\left(1+\frac{1+{}^2/_3 N_D}{1+2N_D}\frac{\langle w'^2\rangle}{u^2}\right)u^2$$

$$\langle v_2'^2\rangle = \langle v_3'^2\rangle = N_v\langle v_1'^2\rangle = \frac{\varphi}{15}\left(1+\frac{1+4N_D}{1+2N_D}\frac{\langle w'^2\rangle}{u^2}\right)u^2$$

$$\langle w_1'v_1'\rangle = \frac{\varphi}{3}\left[l+\frac{3}{5}\left(1+\frac{1+{}^2/_3 N_D}{1+2N_D}\frac{\langle w'^2\rangle}{u^2}\right)\right]u^2 \tag{2.4}$$

$$\langle w_2'v_2'\rangle = \langle w_3'v_3'\rangle = \frac{\varphi}{15}\left(1+\frac{1+4N_D}{1+2N_D}\frac{\langle w'^2\rangle}{u^2}\right)u^2$$

$$\langle \rho' v_j'\rangle = {}^1/_3\varphi(1-\rho)u\delta_{1j}, \quad \langle \rho' w_j'\rangle = (1-\rho)(l+{}^1/_3)u\delta_{1j}$$

The mean values of the products of the components **v'** and **w'** along the different axes are equal to zero, in view of the axial symmetry of the pseudo-turbulent motion.

The dependences of the quantities in (2.4) on ρ have the same character as the dependences $\langle w_j'^2\rangle$, but their maximal values are considerably less. In the approximation $N_D \approx 0$, the ratio $N_v = 1/3$, i.e., the anisotropy of the pulsations of the liquid is much more weakly expressed than the anisotropy of the pulsations of the particles.

Let us evaluate the degree of agreement of the relationships obtained with known experimental results. A large amount of data on the pulsations of a liquid and, in particular, of particles, has been obtained in experiments with fluidized beds. However, unfortunately, the data known to the present author, with respect to local pulsations of very small particles, for which $R < 1$, are very limited and incomplete, and do not inspire any special amount of confidence; this is obviously due to the great experimental difficulties. The minimal sizes of particles whose longitudinal pulsations have been investigated more or less reliably (although still with a large statistical scatter) correspond to values of $R \sim 100$ and higher; in view of this, relationships obtained at $R < 1$ cannot, of course, be considered reliable. If we calculate the value of $\langle w_1'^2 \rangle$ using such relationships, we obtain values which are 1.5-2 times higher than those determined experimentally. Such a comparison has been made, for example, with the data of A. K. Bondareva, given in [5], on the air fluidization of particles of sand with mean diameters of 100, 153, and 233 μ. From a qualitative point of view, the high values of $\langle w_1'^2 \rangle$, calculated in accordance with (2.3), are in the present case completely explainable. Actually, in accordance with (2.3), the value of $\langle w_1'^2 \rangle$ is proportional, roughly speaking, to l^2, where l is determined in (2.2). However, the latter quantity drops rather rapidly with a rise in R; this is explained by the substantial weakening of the dependence of K on ρ with an increase of R [5, 6]. The general character of the dependence of the mean values of the quantities in (2.3), (2.4) on the concentration ρ is completely confirmed experimentally. To ensure a qualitative verification of the theory developed with experimental data, we consider below the effect of a lowering of the hydraulic resistance of a fluidized bed, compared to the resistance of a fixed granular bed of the same particles, and of the same porosity (a detailed discussion of the origin of this effect is given in [2]). The number of investigations devoted to determination of the effective resistance of a fluidized bed at small values of R is very large, and, in contrast to the results of experiments on determination of the pulsations, their results are in rather good agreement with one another (see, for example, the review in [5]).

In accordance with [2] we have (the asterisk denotes the value of K for a fluidized bed)

$$K^*(\rho) = (\lambda_k/\lambda_u)\,K(\rho), \quad K_e^*(\rho) = (\lambda_k/\lambda_u)\,K_e(\rho) \tag{2.5}$$

K^* is connected with K_e^* in the same way as K with K_e (i.e., using the formula (1.5)). The values of λ_k and λ_u are expressed in terms of K, using the relationships

$$\lambda_k = 1 - \varphi\left[l^2 - \frac{(1-\rho)^2}{2K}\frac{d^2K}{d\rho^2}\right], \qquad \lambda_u = 1 - \frac{\varphi}{3} \tag{2.6}$$

The results of a calculation of the value of K^*, carried out in accordance with formulas (2.5) and (2.6) in the region $0.2 < \rho < 0.6$, are given in Fig. 3 (curve 1). The value of K used corresponded to expression (1.6) for K_e; this value of K is shown by the dotted line in Fig. 3. For purposes of comparison, use was made of the empirical dependences of Richardson and Zaki [6, 7] and of Goroshko, Rozenbaum, and Todes [5], describing the hydraulic resistance of a fluidized bed at small values of R, and obtained by analysis of a large number of experimental data. Both of these dependences have the form

$$K_e^* = (1-\rho)^{-n} \tag{2.7}$$

here $n = 4.65$ in the first case, and $n = 4.75$ in the second. Figure 3 gives Curve 2 for K^*, calculated from (2.7) in accordance with (1.5) at $n = 4.7$. We note that dependences of the type of (2.7) have been proposed also in several other articles, in particular in [8], where a value of $n = 4.65$ was also obtained. As is evident from Fig. 3, the agreement between the dependences 1 and 2 for K^* is completely satisfactory, which permits speaking not only of the qualitative, but also, to a certain degree, of the quantitative adequacy of the theory developed.

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